Rossmoyne Senior High School

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

(a) Simplify $\sqrt{9^{-3}}$. (2 marks)

Solution
$\sqrt{9^{-3}} = \left(\sqrt{9}\right)^{-3}$ $= 3^{-3}$ $= \frac{1}{27}$
Specific behaviours
✓ eliminates square root ✓ correct fraction

(b) Write the value of xy in scientific notation when $x = 6 \times 10^6$ and $y = 2.5 \times 10^{-3}$. (2 marks)

Solution
$6 \times 2.5 \times 10^6 \times 10^{-3} = 15 \times 10^3$ $= 1.5 \times 10^4$
Specific behaviours
✓ obtains equivalent expression of form $a \times 10^b$ ✓ correct value using scientific notation

(c) Determine the value of n given that $16^{2n} = \sqrt{32}$. (3 marks)

Solution
$16^{2n} = \sqrt{32}$ $(2^4)^{2n} = \sqrt{2^5}$ $2^{8n} = 2^{\frac{5}{2}}$ $8n = \frac{5}{2}$ $n = \frac{5}{16}$
Specific behaviours
✓ expresses LHS in form 2^a ✓ expresses RHS in form 2^b ✓ correct value of n

Question 2 **(6 marks)**

Solve the following equations.

(a) $16x = 11x + 40.$ (1 mark)

Solution
$5x = 40$ $x = 8$
Specific behaviours
✓ correct solution

(b) $4x^2 = 36x.$ (2 marks)

Solution
$4x^2 - 36x = 0$ $4x(x-9) = 0$ $x = 0, x = 9$
Specific behaviours
✓ equates to 0 and factorises ✓ both correct solutions

0/2 if just x=0

(c) $x^3 + x^2 - 17x + 15 = 0.$ (3 marks)

Solution
When $x = 1$: $\text{LHS} = 1 + 1 - 17 + 15 = 0$ $x^3 + x^2 - 17x + 15 = (x-1)(x^2 + bx - 15)$ $b - 1 = 1 \Rightarrow b = 2$ $(x-1)(x^2 + 2x - 15) = 0$ $(x-1)(x-3)(x+5) = 0$ $x = 1, x = 3, x = -5$
Specific behaviours
✓ indicates that $x - 1$ is a factor/allow other methods ✓ determines two correct solutions ✓ all three solutions(max 2/3 if correct but no supported working)

Question 3 **(6 marks)**

(a) The turning point of a quadratic is at $(-3, -10)$ and the curve passes through $(0, 8)$. Determine the equation of the quadratic in the form $y = ax^2 + bx + c$. (3 marks)

Solution
$y = a(x+3)^2 - 10$ $(0,8) \Rightarrow 8 = 9a - 10$ $a = 2$ $y = 2(x+3)^2 - 10$ $= 2(x^2 + 6x + 9) - 10$ $= 2x^2 + 12x + 8$
Specific behaviours
✓ writes in completed square form using constant ✓ uses y-intercept to evaluate constant ✓ correct equation in required form

(b) Functions f, g and h are defined by $f(x) = 3 + \sqrt{x-5}$, $g(x) = 2f(x)$ and $h(x) = f(x+7)$.

State the

(i) domain of $f(x)$. (1 mark)

Solution
Require $x - 5 \geq 0$: $D_f\ \{x \in \mathbb{R}: x \geq 5\}$ Allow just $x \geq 5$ for all b)
Specific behaviours
✓ states restriction on x

(ii) range of $g(x)$. (1 mark)

Solution
Range of g is $2 \times$ range of f: $R_g\ \{y \in \mathbb{R}, y \geq 6\}$
Specific behaviours
✓ states restriction on y

(iii) domain of $h(x)$. (1 mark)

Solution
h is f translated 7 units left. $D_f\ \{x \in \mathbb{R}: x \geq -2\}$
Specific behaviours
✓ states restriction on x

Question 4 **(6 marks)**

(a) The point $A(1, 2)$ lies on the curve with equation $y = x^3 + 2x^2 - 4x + 3$. Determine the equation of the tangent to the curve at A. (3 marks)

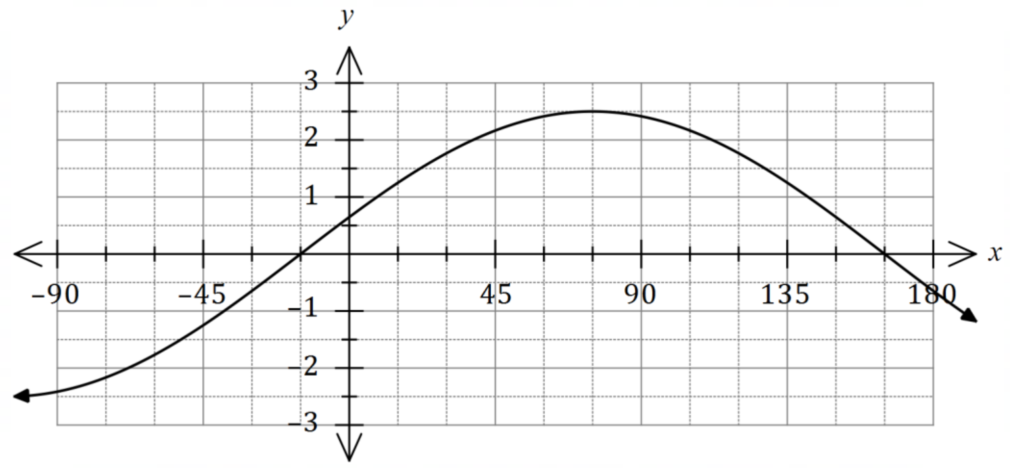
Solution
$$\frac{dy}{dx} = 3x^2 + 4x - 4$$ When $x = 1$ $$\frac{dy}{dx} = 3 + 4 - 4 = 3$$ Equation of tangent: $$y - 2 = 3(x - 1)$$ Or $$y = 3x - 1$$
Specific behaviours
✓ derivative ✓ gradient of tangent ✓ equation of tangent

(b) Determine $g(1)$ given that $g(-1) = 8$ and $g'(x) = 8x^3 + 6x - 7$. (3 marks)

Solution
$$g(x) = 2x^4 + 3x^2 - 7x + c$$ Using $g(-1) = 8$: $$2 + 3 + 7 + c = 8$$ $$c = -4$$ $$g(1) = 2 + 3 - 7 - 4$$ $$= -6$$
Specific behaviours
✓ antiderivative ✓ determines constant ✓ correct value 0/3 if no c value

Question 5 **(7 marks)**

(a) Part of the graph of $y = a\cos(x - \theta)$ is shown below.



State the value of the constant a and the value of the constant θ, $0° \leq \theta \leq 180°$.

(2 marks)

Solution
$a = 2.5, \quad \theta = 75°$
Specific behaviours
✓ correct amplitude ✓ correct phase angle

(b) i) Show that $\cos(x + y) + \cos(x - y) = k\cos x\cos y$ and state the value of the constant k.

(2 marks)

Solution
$\cos(x+y) + \cos(x-y) = \cos x\cos y + \sin x\sin y + \cos x\cos y - \sin x\sin y$ $= 2\cos x\cos y$ $k = 2$
Specific behaviours
✓ uses sum and difference identities correctly ✓ states correct value of k

ii) Hence or otherwise determine an exact value for $\cos 75° + \cos 15°$. (3 marks)

Solution
If $x = 45°$ and $y = 30°$ then $x + y = 75°$ and $x - y = 15°$. Hence $\cos 75° + \cos 15° = 2\cos 45°\cos 30°$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$ $= \frac{\sqrt{6}}{2}$
Specific behaviours
✓ indicates suitable values for x and y ✓ uses correct exact values ✓ correct, simplified surd (allow $0.5\sqrt{2}\sqrt{3}$)

Question 6 **(7 marks)**

(a) A sequence is defined by $T_{n+1} = T_n + 0.3,\ T_1 = 5$. Determine

(i) T_{101}. (2 marks)

Solution
$T_{101} = 5 + (100)(0.3)$ $= 35$
Specific behaviours
✓ indicates use of general term formula ✓ correct term

(ii) the sum of the first 101 terms of the sequence. (2 marks)

Solution
$S_{101} = \frac{101}{2}\big(2(5) + (100)(0.3)\big)$ $= \frac{101 \times 40}{2} = 101 \times 20 = 2020$
Specific behaviours
✓ indicates correct use of sum formula ✓ correct sum

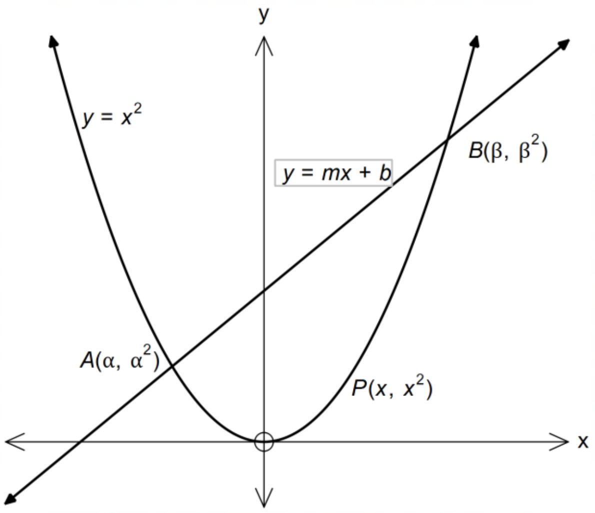
or $S_{101} = \frac{101}{2}(5 + 35)$

(b) The sum to infinity of the series $4 + 4k + 4k^2 + 4k^3 + \cdots$ is 10. Determine the sum of the first three terms of the series. (3 marks)

Solution
Series is geometric with $a = 4$ and $r = k$. $\frac{4}{1-k} = 10$ $4 = 10 - 10k$ $10k = 6$ $k = \frac{3}{5} = 0.6$ $S = 4 + 4(0.6) + 4(0.6)^2$ $= 4 + 2.4 + 1.44$ $= 7.84$ NB $S = 4 + 4\left(\frac{3}{5}\right) + 4\left(\frac{3}{5}\right)^2 = 4 + \frac{12}{5} + \frac{36}{25} = \frac{100 + 60 + 36}{25} = \frac{196}{25}$
Specific behaviours
✓ equation using sum to infinity ✓ value of k ✓ correct sum

Question 7 **(6 marks)**

The parabola $y = x^2$ intersects with the line AB at the points $A\left(\alpha,\ \alpha^2\right)$ and $B\left(\beta,\beta^2\right)$ as shown in the diagram.



(a) Show clearly that . (4 marks)

Solution
$m = \dfrac{\beta^2 - \alpha^2}{\beta - \alpha} = \dfrac{(\beta + \alpha)(\beta - \alpha)}{\beta - \alpha}$ $\therefore m = \alpha + \beta$ $y = (\alpha + \beta)x + b \ \mid \left(\alpha,\ \alpha^2\right)$ $\alpha^2 - \alpha^2 - \alpha\beta = b$ $\alpha\beta = -b$
Specific behaviours
✓ Shows use gradient formula using points *A* and *B*   ✓ factorises numerator and simplifies correctly   ✓ Substitutes *m*= *α*+*β* into general equation line   ✓ Substitutes either point A or B to simplify correctly

(b) Given that α = - 2 and that P is a point on the parabola such that the midpoint of the line segment AP is $(-0.25,\ 3.125)$. Determine the co-ordinates of the point P. (2 marks)

Solution
$\dfrac{-2 + x}{2} = -0.25$ $x = 1.5$ $y = 1.5^2 = 2.25$ (or $\dfrac{4 + y}{2} = 3.125$ $y = 2.25$) (1.5, 2.25)
Specific behaviours
✓ correct x value   ✓ correct y value (-1 if not written as co-ordinates)

Question 8 **(7 marks)**

The line $y = 6x + c$ is a tangent to the curve $y = 2x^3 + 3x^2 - 6x - 3$. Determine the value(s) of the constant c.

Solution
Gradient of cubic: $$\frac{dy}{dx} = 6x^2 + 6x - 6$$ Gradient of line is 6 so: $$6x^2 + 6x - 6 = 6$$ $$6x^2 + 6x - 12 = 0$$ $$x^2 + x - 2 = 0$$ $$(x-1)(x+2) = 0$$ $$x = 1, x = -2$$ At $x = 1$: $$y = 2 + 3 - 6 - 3 = -4$$ $$y + 4 = 6(x - 1)$$ $$y = 6x - 10 \Rightarrow c = -10$$ At $x = -2$: $$y = -16 + 12 + 12 - 3 = 5$$ $$y - 5 = 6(x + 2)$$ $$y = 6x + 17 \Rightarrow c = 17$$ Hence $c = 17,\ c = -10$.
Specific behaviours
✓ gradient function for cubic ✓ equates $\frac{dy}{dx} = 6$ ✓ simplifies and factors quadratic ✓ both solutions to quadratic ✓ y-coordinate of point of tangency ✓ one value of c ✓ repeats for second value of c

Supplementary page

Question number: _________